A Mathematical Model of Circumstellar and Circumplanetary Habitable Zones Accounting for Multiple Heat Sources

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Abstract

The habitable zone is the range of orbital radii at which a celestial body's temperature is conducive to liquid water. The ability to predict exoplanetary habitability allows for understanding of the range of orbital parameters in which life as we know it may arise and survive. In this paper, elements of mathematical models in the literature developed to determine energy contributions by stellar radiation and tidal heating were combined, producing a Maple 18 model accounting for both circumstellar and circumplanetary habitability. Using input star, planet, and moon properties, the model outputs calculated temperatures for both the planet and moon, as well as coloured maps representing the habitable zone. Using this model, the effect on the habitable zone when tidal heating is considered in addition to stellar radiation was assessed, giving a better understanding of how these factors influence habitability. Specifically, tidal heating pushes both edges of the habitable zone outward, but overall, tends to decrease the size of the habitable zone. However, accounting for the circumplanetary habitable zone in addition to the circumstellar habitable zone expands the overall habitable zone. By synthesizing a single model from existing literature models, a more accurate prediction of exoplanetary system habitability may be obtained, allowing scientists to better determine which exoplanets are more likely to have life and target these for further study. Therefore, the model was applied to existing exoplanet data to demonstrate this application of the model in predicting exoplanet and exomoon temperatures.

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Introduction

The relatively new field of astrobiology is concerned with the origins, evolution, distribution, and future of life in the universe (National Aeronautics and Space Administration, 2014a). An important aspect of this field is understanding the range of conditions under which life as we know it can develop and exist (Des Marais, et al., 2008). One condition often considered is the presence of liquid water: for life as we know it, water is the universal solvent, and its polar nature grants it many properties that are useful to life (Mottl, et al., 2007). Thus, the "habitable zone" around a star is often defined as the set of orbital radii around a star that allows liquid water to exist on a celestial body (Doyle, Billingham and DeVincenzi, 1998). Traditionally, most calculations of habitable zones have only considered the amount of energy received by a planet due to its star's illumination: while this is a major component for assessing habitability, other factors are also involved (Kaltenegger and Segura, 2011). One of the most intriguing of these additional factors is tidal heating. Tidal heating is the heat generated by the flexing of a celestial body's surface due to the gravitational forces exerted by an orbiting body. While the heat generated by the moon's tidal heating of the Earth's oceans is negligible compared to the incoming stellar radiation, tidal heating is considered a major source of energy for celestial bodies like Jupiter's moon Europa (Patiño Douce, 2011). Tidal heating by

Jupiter's gravity delivers 6TW of power to Europa, which is enough to generate what is believed to be a subsurface liquid water ocean on Europa (Greenberg, 2007).

In this paper, I produce a model that takes into account both solar illumination and tidal heating as a sources of heat for a celestial body. This model considers both planets and moons together, whereas previous studies have generally considered one or the other in isolation. Thus, this paper works to combines previous work including Heller and Barnes (2013), who consider exomoons alone, and Jackson, Barnes and Greenberg (2008), who examine only tidal heating of planets.

Factors Affecting Habitability

In this work, the habitable zone refers to the set of regions in space around a star in which a celestial body receives enough heat energy for liquid water to exist. Thus, factors affecting habitability are the various sources of heat that a celestial body may have.

As noted above, stellar illumination is one of the greatest factors and is most commonly used to calculate habitable zones. Another factor affecting the amount of heat energy a celestial body receives is its atmosphere. The atmosphere may reflect some of the incoming stellar radiation, for example, due to clouds. This, combined with the light reflected by the body's surface, is quantified as the albedo. However, the atmosphere can also help the planet retain the heat it absorbs through the greenhouse effect (Kaltenegger and Segura, 2011).

Besides the factors involving stellar radiation (stellar illumination, albedo, and the greenhouse effect), tidal heating is another factor that can affect habitability. Tidal heating occurs due to the flexing of a celestial body's surface caused by gravitational attraction by an orbiting body. This heating can occur through multiple mechanisms. When the orbiting bodies are not tidally locked (they do not rotate at the same speed and so do not always present the same face to one another), the tidal bulge raised on one body by the other rotates away as the first body rotates. This causes the bulge to "travel" across the surface in order to continue facing the other body; the required deformation of material causes the release of heat. However, tidal heating can also occur for tidally locked systems. When one body is tidally locked to another, the same face is always facing the other body. Thus, the bulge does not travel across the surface and no tidal heat is generated through the first mechanism. However, if the tidally locked body's orbit is eccentric, then at different points in its orbit, it will be at different distances to the second body. This means it will experience varying levels of the gravitational force over its orbit, so the tidal bulge will change in height at different points in the orbit. Again, this change in height requires the deformation of material, so heat is released in the process (Patiño Douce, 2011). Another tidally-induced form of heating involves Rossby waves, which are oceanic waves with long wavelength and low amplitude that can transfer energy over distances (Tyler, 2008).

In addition, a celestial body can also gain heat from other sources. For example, heat can be released by the radioactive decay of radionuclides in a body's interior (Frank, Meyer and Mojzsis, 2014). As well, there are various types of heat associated with planetary formation including heat from accretion, gravitational compression, and core formation (Horedt, 1980).

Materials and Methods

Factors Considered in the Model

While it would be ideal to consider all the factors described previously, this is not feasible due to the dependence of many of these factors on parameters for which data is not widely available. Thus, this model considers only stellar radiation and tidal heating. While the greenhouse effect plays a major role on planets with substantial atmospheres, it is difficult to model atmospheric effects, so this factor is neglected. Furthermore, calculations of radiogenic and primordial heat require knowledge of the physical composition of celestial bodies, and exact quantitative data on this is not easily accessible, especially for exoplanets and exomoons.

Tidal heating, as noted above, consists of various different components. Attempts were made to include all components discussed, but the mathematics behind Rossby waves were too complex to incorporate in the available timeframe. Additionally, they require the assumption of a fluid covering the surface of the celestial body, an assumption that would be in contrast to other assumptions made while calculating the other components of tidal heating, namely, the assumption of a solid, Earth-like surface (see the Assumptions section below).

Mathematical Equations for the Factors

An equation quantifying the amount of heat contributed by a particular factor is necessary in order to be able to incorporate that factor into the mathematical model. In order to describe the amount of energy received by a celestial body per unit time (power), we use the equation

$$\pi R_{Planet}^2(1\text{-}A)F_a=4\pi R_{Planet}^2\sigma T_{Planet}^4 \qquad \textbf{(1)}$$

where A is the (unitless) albedo of the planet, F_a is the flux at the given orbital radius a in W/m^2, R is the radius of the planet (or moon), σ is the Stefan-Boltzmann constant, and T is the temperature of the celestial body in Kelvin — the value being solved for (Seager, 2010). The equation can be understood in physical terms as describing the celestial body in a state of thermal equilibrium where the incoming heat (left side) is balanced by the heat being lost (right side). On the left side, the incoming flux is F_a. This is multiplied by the area of the celestial body facing the star, which appears as a disc and is therefore represented by πR^2. This value can then be multiplied by the (1-A) term to account for incoming energy that is not absorbed due to albedo. On the left side, the entire surface of the celestial body is assumed to radiate heat equally, so the full surface area of a sphere, $4\pi R^2$, is used. This is multiplied by σT^4, which is equal to the flux leaving the planet (as stated by the Stefan-Boltzmann law).

To calculate the stellar flux at a given distance, a second formula is required:

$$F(a)=R_{Star}^2\sigma T_{Star}^4/a^2 \qquad \textbf{(2)}$$

where R_{Star} is the radius of the star, T_{Star} is the temperature of the star, a is the semi-major axis length in meters, and F is the flux as a function of the semi-major axis length, in W/m^2 (Seager, 2010). This formula can be interpreted as an expression of the conservation of energy: a star radiates $4\pi R_{Star}^2\ \sigma T_{Star}^4$ of power (surface area $4\pi R_{Star}^2$ multiplied by flux σT_{Star}^4), which is conserved as it moves to greater radii, so the same amount of energy is spread out over a larger surface area, $4\pi a^2$. The 4π cancels out, and the above expression is obtained. This equation also works for stellar radiation falling on a moon if R_{Moon} and T_{Moon} are used instead.

There are many different formulae used to calculate the tidal heat. Here, four formulae are compared, from Heller and Barnes (2013), Henning, O'Connell and Sasselov (2009), Jackson, Barnes and Greenberg (2008), and Greenberg (2007), with additional information from Carone and Pätzold (2007) and the National Aeronautics and Space Administration (2014b, 2014c), along with the authors' estimates of the remaining parameters (see Appendix A). It was found that the calculated tidal energy was all of the same order of magnitude, indicating that all models are fairly good. However, the model by Heller and Barnes (2013) gave the value closest to the 6TW figure calculated by Greenberg (2007) by scaling Europa's physical and orbital parameters to those of Io, for which the tidal heating has been estimated through measurement. Thus, the model by Heller and Barnes (2013) was selected:

$$E = \frac{3G^2 k_{2,s} M_p^2 \left(M_p + M_s\right) \frac{R_s^5}{a_s^9} \tau_s}{\sqrt{1 - e_{ps}^2}^{15}} \left[\left(1 + \frac{32}{2} e_{ps}^2 + \frac{255}{8} e_{ps}^4 + \frac{185}{16} e_{ps}^6 + \frac{25}{64} e_{ps}^8\right) - \frac{1 + \frac{15}{2} e_{ps}^2 + \frac{45}{8} e_{ps}^4 + \frac{5}{16} e_{ps}^6}{1 + 3e_{ps}^2 + \frac{3}{8} e_{ps}^4} \right] \quad \textbf{(3)}$$

where G is the gravitational constant, $k_{2,s}$ is the second-order potential Love number (a dimensionless measure of the rigidity of a celestial body), M_p is the mass of the parent body (the planet in a moon-planet interaction or the star in a star-planet interaction), M_s is the mass of the satellite, a_{ps} is the orbital radius (semi-major axis) of the satellite about the parent body, R_s is the radius of the satellite, τ_s is the constant time lag factor (the duration of time lag for the bulge to reach its equilibrium position (Hut, 1981), e_{ps} is the eccentricity of the satellite's orbit around the parent body, and E is the energy released due to tidal heating per unit time, in watts.

Parameters Required for the Model

Based on the above equations, the parameters that the model would require are listed in the table below:

Table 1: Parameters required for the habitable zone model calculations

Star	Planet	Moon
Radius	Radius	Radius
Temperature		
Mass	Mass	Mass
	Circumstellar semi-major axis	Circumplanetary semi-major axis
	Eccentricity	Eccentricity
	Second-order Love number	Second-order Love number
	Time lag constant	Time lag constant

Most of these parameters can easily be found for any given planet/moon in the solar system, and for several exoplanetary systems too. However, the Love number and time lag constant are based on physical properties of the bodies and are empirically determined. Thus, in this paper, they are approximated using known values from the Earth, meaning that the planets and moons are assumed to have an Earth-like composition. Similarly, the albedo of the planet is assumed to be equal to that of the Earth for the purposes of calculation, since data for this only exists for planets that can be directly observed, i.e., planets within this solar system, and not for exoplanets.

Assumptions

As with most mathematical models, assumptions must be made in order for the system to be describable in mathematical terms. In this model, various factors are neglected due to feasibility. First of all, the greenhouse effect is neglected: this normally has a fairly large effect on the habitability of a planet. For example, Venus' greenhouse effect makes it hundreds of Kelvin warmer than predicted by solar radiation alone (Lang, 2011). As well, in tidal heating interactions, both bodies exert tidal forces on one another (Patiño Douce, 2011), but the tidal force of the satellite on the parent body is neglected since it will be much smaller than the effect the parent body has on the satellite, unless the satellite is close in size to the parent body. As well, it is neglected that in these interactions, the eccentricity and semi-major axis length change over time because the bodies slow each other down, and conservation of angular momentum leads the bodies to increase in distance from one another over time, which would change the magnitude of the tidal force over time (Heller and Barnes, 2013). Another limitation of the model used is that it does not account for Rossby wave tidal heating, which has been calculated to contribute a large portion of heat which is orders of magnitude greater than radiogenic heating and the types of tidal heating included in this model (Tyler, 2008). Finally, it is assumed that an exomoon receives the same stellar flux as its parent body, neglecting the fact that it is likely for the parent body to block stellar radiation from reaching the satellite for some portion of the satellite's orbit (Heller and Barnes, 2013).

Implementation of Model and Visualization

The combined equation from above was input into the computer algebra software Maple 18 (Maplesoft, 2014). A code was then developed to associate different temperatures with different colours. In this model, the freezing and melting points of water were used as the thresholds for habitability. Temperatures greater than the boiling point of water were mapped to red colours. Temperatures from the boiling point of water to the stellar temperature were mapped linearly to the intensity of the red colour, with the most intense red colour corresponding to 1010K, and the boiling point having the least intense red colour. Likewise, temperatures from 0K to the melting point of water ice were mapped linearly to the colour blue, with the most intense blue corresponding to absolute zero. Temperatures considered habitable were green, with the most intense green corresponding to the temperature in the middle of the habitable range, with the intensity of colour decreasing linearly towards both edges.

Initially, the model was intended to superimpose a circumstellar habitable zone (accounting for stellar radiation and stellar tidal heating on planets) with the circumplanetary habitable zone (accounting for stellar radiation and planetary tidal heating of moons) for the given planetary and moon data. However, it turned out that the circumplanetary habitable zone was extremely small relative to the circumstellar habitable zone, such that except at extreme orbital eccentricities, the circumplanetary habitable zone was almost invisible (Figure 1).

Therefore, the model was redesigned to superimpose a circumstellar habitable zone for planets, based on stellar radiation and tidal heating of the planet by the star, with a circumstellar habitable zone for moons, based on stellar radiation and tidal heating of the moon by the planet, given a certain circumplanetary orbital semi-major axis of the moon.

Superimposition was accomplished by the following precedence rules, applied in the given order:

1. Habitable zones
2. "Hot" zones
3. Planetary temperature
4. Moon temperature

The results of applying these rules are summarized in the table below, where the subscript letters refer to the planet or the moon being the source of the temperature used to calculate the colour.

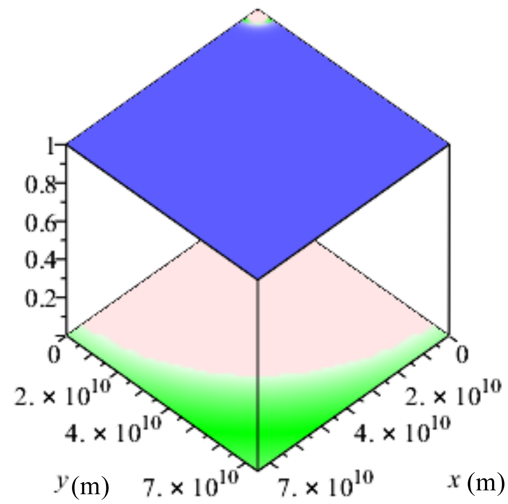


Figure 1: A comparison of the circumstellar planetary habitability with the circumplanetary moon habitability, using data from the Sun-Jupiter-Europa system, with Europa's eccentricity increased to 0.94 from its regular value of 0.0094. The lower surface corresponds to the habitable zone of a Jupiter with the Sun at the origin, while the top surface corresponds to the habitable zone for a Europa, where the origin is Jupiter. Image produced using (Maplesoft, 2014). Green regions correspond to habitable temperatures; blue and red represent orbital distances that are too cold and too hot, respectively.

Table 2: Precedence rules used for determining the temperature used to calculate the colour of a point on the map

		Planet		
		Cold	Habitable	Hot
	Cold	Blue$_P$	Green$_P$	Red$_P$
Moon	Habitable	Green$_M$	Green$_P$	Green$_M$
	Hot	Red$_M$	Green$_P$	Red$_P$

The complete mathematical model can be found in Appendix B.

Results and Discussion

Although stellar illumination is the dominant factor in determining the habitable zone for planets, there are other factors involved. In this model, tidal heating is also considered: tidal heating provides additional heat to the planet, pushing the outer boundary of the habitable zone away. At the same time, the inner edge also moves outward due to the additional heat energy. What is not immediately evident is whether the orbital range of the habitable zone changes. It has been suggested that tidal heating shrinks the habitable zone (Barnes, et al., 2012). This theory is supported by the equations used in this model, as stellar radiation energy decreases with the square of distance to the star (a_{planet}), while tidal heating decreases with the ninth power of distance to the star (a_{planet}). This would suggest that the inner edge of the habitable zone should be pushed out a greater amount than the outer edge, shrinking the habitable zone. To ascertain that this was indeed the case, the equations used in the model were solved to find the orbital semi-major axis lengths where the planet temperature was equal to the freezing and melting points of water. As the results in Appendix C show, increased eccentricity, which corresponds to increased tidal heating, shrinks the habitable zone range.

As can be expected, the superimposition method used in this paper, where habitability of either the planet or moon was sufficient for an orbital radius to be considered habitable, increased the overall habitable zone. However, this effect is not very noticeable at low moon eccentricities like those that are found for most moon-planet systems in this solar system (Figure 2).

By synthesizing a model from multiple sources in the existing literature, we are able to consider a wider range of factors. This allows us to more accurately predict the habitability of an exoplanet (and potential exomoon) using data available through today's technologies. To demonstrate this, the model is applied to the Sun/Jupiter/Europa system, as well as the Kepler-22/Kepler-22b system (no exomoons have yet been detected due to limitations of technology). Figure 3 shows that the Jupiter-Europa system falls outside of the habitable zone: a temperature of 110.65K is calculated for Jupiter, and a temperature of 111.26K is calculated for Europa. These temperatures are far below the freezing point of water at atmospheric pressure, indicating that the presence of liquid water is unlikely.

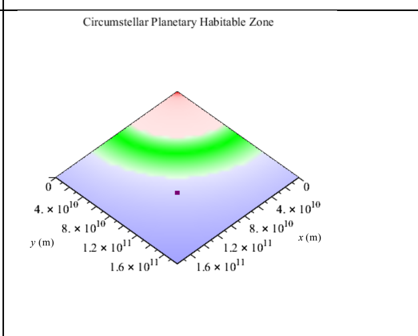
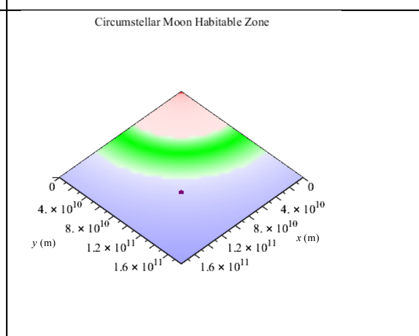
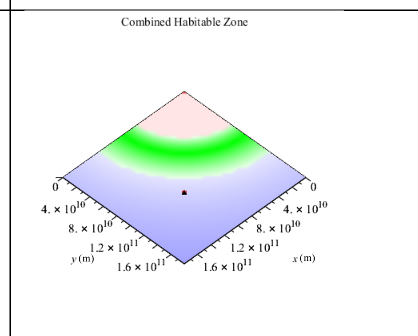
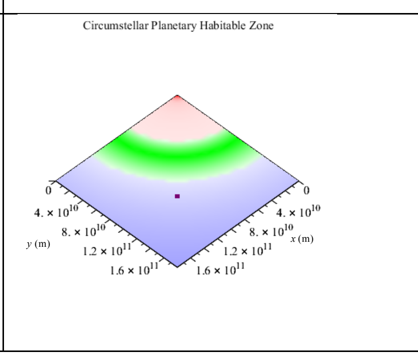
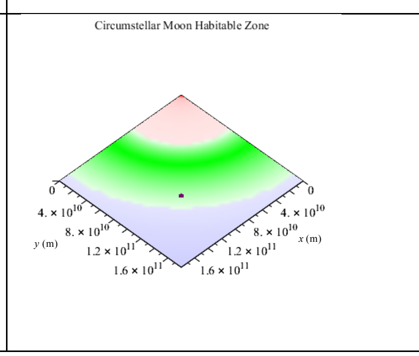
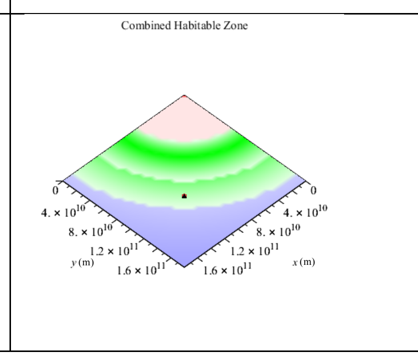
Eccentricity	Circumstellar Planetary HZ	Circumstellar Moon HZ	Combined HZ
0.04			
0.2			

Figure 2: A Jupiter-sized planet with a Europa-sized moon, with the planet orbiting at 1AU from the star. The planet has a constant eccentricity of 0.04, while the moon has eccentricity 0.04 in one row, and an eccentricity of 0.2 in the second row. In the system with eccentricity of 0.04, the combined habitable zone is not visibly different from that of either the planet or moon. At an eccentricity of 0.02, the combined habitable zone is visibly larger than the planetary habitable zone alone. As can be seen, the system, represented by the black dot, normally too cold for life, falls into the habitable zone due to tidal heating if the moon's eccentricity is high enough. Image generated using (Maplesoft, 2014). Colour scheme is the same as in Figure 1, and as described in the Methods. All axes are on the same scale.

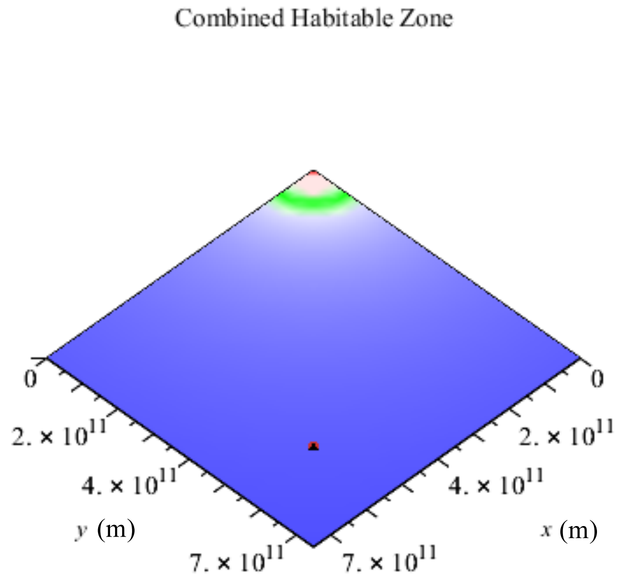


Figure 3: The combined habitable zone for the Sun-Jupiter-Europa system. Jupiter and Europa both clearly fall outside the habitable zone. Colour scheme is the same as in Figure 1, and as described in the Methods.

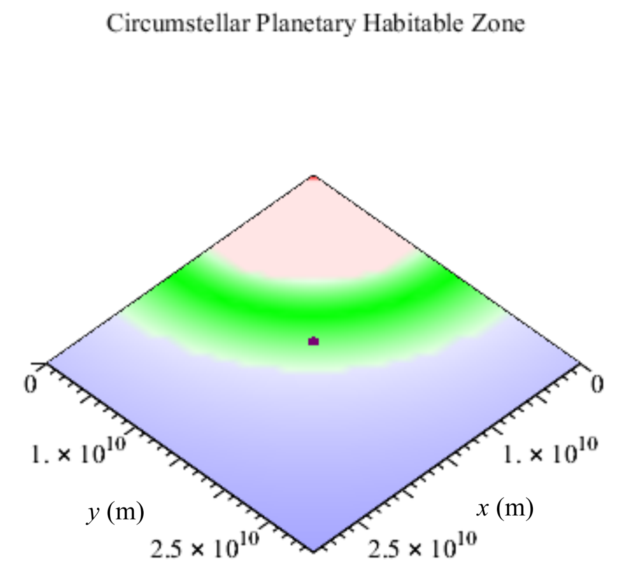


Figure 4: A plot of the Kepler-22 and Kepler-22b system. Kepler-22b apparently lies within the star's habitable zone. Colour scheme is the same as in Figure 1, and as described in the Methods.

For Kepler-22b (Figure 4), the calculated temperature is 295.84K, indicating it could be habitable.

The calculated temperature for Europa is much colder than the freezing point of water, despite evidence suggesting that Europa has a liquid water ocean. Likewise, when the model is run on the Earth, it predicts a temperature on Earth lower than the freezing point of water (see Appendix B). These results are indicative of the simplifications made in this model. For instance, Rossby waves are expected to contribute greatly to Europa's heat, while the Earth is kept warmer than predicted due to the greenhouse effect.

Conclusion

In this paper, a model of circumstellar planetary and moon habitable zones was produced by combining aspects of previous work in the literature. The resulting model accounts for both stellar radiation and tidal heating when calculating the habitable zone around a star for both a planet and for a moon orbiting the planet at a given circumplanetary radius. Through the calculations used in the model, it was shown that the addition of tidal heating shrinks habitable zones compared to those calculated by models that account only for stellar radiation. However, because moons may be tidally heated to temperatures allowing liquid water at circumstellar radii where water would normally be frozen, the model also demonstrates that the habitable zone may extend further than previously thought. However, the model bears some weaknesses due to several simplifying assumptions made to avoid factors that are difficult to calculate using available astronomical data. Nevertheless, this model is a good starting place for gaining a more accurate understanding of the habitable zone around a star. Future work could build on this by adding more factors into the calculations.

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Appendix A – Comparison of Tidal Heating Models

We first define the variables: G is the gravitational constant, k is k_2, the second-order potential Love number, Mp is the mass of the parent body, Ms is the mass of the satellite, a is the semi-major axis of the orbit of the satellite around the parent body, τ is the time lag constant, Rs is the radius of the satellite, Q is the quality factor, Qk is a dissipation factor that takes into account the Love number. Values were drawn from Heller and Barnes (2013), Henning, O'Connell and Sasselov (2009), Jackson, Barnes and Greenberg (2008), and the National Aeronautics and Space Administration (2014b, 2014c, 2014d). In addition, a term called n, the mean orbital motion, was expanded in terms of these parameters by a formula given in Carone and Pätzold (2007).

The models were tested in the following order: Heller and Barnes (2013), Henning, O'Connell and Sasselov (2009), Jackson, Barnes and Greenberg (2008), then Greenberg (2007). The output is the tidal heating energy in watts. Calculations were performed on Maple 18 (Maplesoft, 2014).

> $G := 6.67384 \cdot 10^{-11}; k := 0.3; Mp := 1.89813 \cdot 10^{27}; Ms := 4.79984383874927 \cdot 10^{22}; a := 671100000; \mathrm{tau} := 638; Rs := 1560800; Q := 50; Qk := 500; e := 0.0094$

$$G := 6.673840000\,10^{-11}$$
$$k := 0.3$$
$$Mp := 1.898130000\,10^{27}$$
$$Ms := 4.799843839\,10^{22}$$
$$a := 671100000$$
$$\tau := 638$$
$$Rs := 1560800$$
$$Q := 50$$
$$Qk := 500$$
$$e := 0.0094$$

> $$\frac{3 \cdot G^2 \cdot k \cdot Mp^2 \cdot (Mp + Ms) \cdot Rs^5}{a^9} \cdot \frac{\mathrm{tau}}{(1-e)^{\frac{15}{2}}} \left(\left(1 + \frac{31}{2} \cdot e^2 + \frac{255}{8} \cdot e^4 + \frac{185}{16} \cdot e^6 + \frac{25}{64} \cdot e^8\right) - \frac{\left(1 + \frac{15}{2} \cdot e^2 + \frac{45}{8} \cdot e^4 + \frac{5}{16} \cdot e^6\right)}{\left(1 + 3 \cdot e^2 + \frac{3}{8} e^4\right)} \right)$$

$$6.123582051\,10^{12}$$

> $$\frac{21}{2} \cdot \frac{k}{Q} \cdot \left(\frac{G \cdot Mp^2 \cdot Rs^5 \cdot \mathrm{sqrt}\left(\frac{G \cdot (Ms + Mp)}{a^3}\right) \cdot e^2}{a^6} \right)$$

$$2.778512197\,10^{12}$$

> $$evalf\left(\frac{63}{4} \cdot \frac{(G \cdot Mp)^{\frac{3}{2}} \cdot Mp \cdot Rs^5}{Qk} \cdot a^{-\frac{15}{2}} e^2 \right)$$

$$1.389238534\,10^{12}$$

> $$\frac{21}{2} \frac{Rs^5 \cdot e^2 \cdot \left(\frac{G \cdot (Ms + Mp)}{a^3}\right)^{\frac{5}{2}} \cdot k}{G \cdot Q}$$

$$2.778652722\,10^{12}$$

Appendix B – Complete Habitable Zone Model

The following is the completed mathematical model, using numbers for the Sun-Earth-Moon system from the National Aeronautics and Space Administration (2014b, 2014e, 2014f).

Star Data

Here, you can input or adjust the stellar parameters: these are its radius (in metres), temperature (in Kelvin), and mass (in kilograms)

$Rstar := 695800000:$

$Tstar := 5778:$

$Mstar := 1.9891 \cdot 10^{30}:$

Planet Data

Here, you can input or adjust the planetary parameters: these are its radius (in metres), mass (in kilograms), semimajor axis length of its circumstellar orbit (in metres), and eccentricity of said orbit (unitless):

$Rplanet := 6371000:$

$Mplanet := 5.9722 \cdot 10^{24}:$

$aplanet := 149598262000:$

$eplanet := 0.01671123:$

Moon Data

Here, you can input or adjust the planetary parameters: these are its radius (in metres), mass (in kilograms), semimajor axis of its circumplanetary orbit (in metres), and eccentricity of said orbit (unitless):

$Rmoon := 1737500:$

$Mmoon := 7.3477 \cdot 10^{22}:$

$amoon := 384400000:$

$emoon := 0.0554:$

Constants

Here are several constants that may be changed. AU is the maximum circumstellar orbital distance along either the x or y axis you wish to evaluate the colour map to (in astronomical units), sigma is the Stefan-Boltzmann constant (in $\frac{W}{m^2 K^4}$), G is the gravitational constant (in $\frac{m^3}{kg \cdot s^2}$), k is the second-order Love number of both the planet and moon (unitless), tau is the constant time lag of the tidal bulge (in seconds), A is the albedo of the planet and moon (unitless), and Digits defines how many digits are used in the calculations.

$AU := 1.2:$

$\text{sigma} := 5.670373 \cdot 10^{-8}:$

$G := 6.67384 \cdot 10^{-11}:$

$k := 0.3:$

$\text{tau} := 638:$

$A := 0.33:$

$Digits := 20:$

Calculation

What follows it the code of the calculation. It will return the planet's temperature, the moon's temperature, separate graphs of the circumstellar and circumplanetary habitable zones, and a superimposition of the two graphs.

$$planettemp := (dist) \rightarrow \Bigg(\Bigg(Rplanet^2 \cdot 0.52 + \frac{\text{Pi} \cdot Rplanet^2 \cdot (1 - A) Rstar^2 \cdot \text{sigma} \cdot Tstar^4}{dist^2}$$

$$+\frac{3\cdot G^{2}\cdot k\cdot Mstar^{2}\cdot(Mstar+Mplanet)\cdot Rplanet^{5}\cdot \mathrm{tau}}{dist^{9}\cdot \mathrm{sqrt}(1-eplanet^{2})^{15}}\left(\left(\left(1+\frac{31}{2}eplanet^{2}+\frac{255}{8}eplanet^{4}+\frac{185}{16}eplanet^{6}+\frac{25}{64}eplanet^{8}\right)-\left(1+\frac{15}{2}\cdot eplanet^{2}+\frac{45}{8}\cdot eplanet^{4}+\frac{5}{16}\cdot eplanet^{6}\right)\cdot\left(1+3\,eplanet^{2}+\frac{3}{8}eplanet^{4}\right)^{-1}\right)\right)\cdot(4\cdot \mathrm{Pi}\cdot Rplanet^{2}\cdot \mathrm{sigma})^{-1}\Bigg)^{\frac{1}{4}}:$$

$$Planet\ Temperature = evalf(planettemp(aplanet))\ K;$$

$$moontemp := (dist)\rightarrow\left(\left(Rmoon^{2}\cdot 0.52+\frac{\mathrm{Pi}\cdot Rmoon^{2}\cdot(1-A)Rstar^{2}\cdot \mathrm{sigma}\cdot Tstar^{4}}{dist^{2}}+\frac{3\cdot G^{2}\cdot k\cdot Mplanet^{2}\cdot(Mplanet+Mmoon)\cdot Rmoon^{5}\cdot \mathrm{tau}}{amoon^{9}\cdot \mathrm{sqrt}(1-emoon^{2})^{15}}\left(\left(\left(1+\frac{31}{2}emoon^{2}+\frac{255}{8}emoon^{4}+\frac{185}{16}emoon^{6}+\frac{25}{64}emoon^{8}\right)-\left(1+\frac{15}{2}\cdot emoon^{2}+\frac{45}{8}\cdot emoon^{4}+\frac{5}{16}\cdot emoon^{6}\right)\cdot\left(1+3\,emoon^{2}+\frac{3}{8}emoon^{4}\right)^{-1}\right)\right)\cdot(4\cdot \mathrm{Pi}\cdot Rmoon^{2}\cdot \mathrm{sigma})^{-1}\right)^{\frac{1}{4}}:$$

$$plot3d\left(\left[0,\left[\frac{aplanet}{\mathrm{sqrt}(2)},\frac{aplanet}{\mathrm{sqrt}(2)},0\right]\right],x=0..AU\cdot 149597871000,y=0..AU\cdot 149597871000,color=\left[\left[piecewise\left(planettemp(\mathrm{sqrt}(x^{2}+y^{2}))<273,\frac{planettemp(\mathrm{sqrt}(x^{2}+y^{2}))}{273}\cdot 0.9,273\le planettemp(\mathrm{sqrt}(x^{2}+y^{2}))\le 373,\left(\frac{\mathrm{abs}(323-planettemp(\mathrm{sqrt}(x^{2}+y^{2})))}{50}\cdot 0.9\right),planettemp(\mathrm{sqrt}(x^{2}+y^{2}))>373,1\right),piecewise\left(planettemp(\mathrm{sqrt}(x^{2}+y^{2}))<273,\frac{planettemp(\mathrm{sqrt}(x^{2}+y^{2}))}{273}\cdot 0.9,273\le planettemp(\mathrm{sqrt}(x^{2}+y^{2}))\le 373,1,planettemp(\mathrm{sqrt}(x^{2}+y^{2}))>373,\frac{(10^{10}-planettemp(\mathrm{sqrt}(x^{2}+y^{2})))}{10^{10}-373}\cdot 0.9\right),piecewise\left(planettemp(\mathrm{sqrt}(x^{2}+y^{2}))<273,1,273\le planettemp(\mathrm{sqrt}(x^{2}+y^{2}))\le 373,\left(\frac{\mathrm{abs}(323-planettemp(\mathrm{sqrt}(x^{2}+y^{2})))}{50}\cdot 0.9\right),planettemp(\mathrm{sqrt}(x^{2}+y^{2}))>373,\frac{(10^{10}-planettemp(\mathrm{sqrt}(x^{2}+y^{2})))}{10^{10}-373}\cdot 0.9\right)\right],purple\right],numpoints=[1000,1],lightmodel=none,style=[surface,point],symbolsize=20,scaling=constrained,title=\text{"Circumstellar Planetary Habitable Zone"},labels=[\text{"Distance (m)"},\text{""},\text{""}],view=0..0.1\right);$$

$$Moon\ Temperature = evalf(moontemp(aplanet))\ K;$$

$$plot3d\left(\left[0, \left[\frac{aplanet}{\mathrm{sqrt}(2)} - \frac{amoon}{\mathrm{sqrt}(2)}, \frac{aplanet}{\mathrm{sqrt}(2)} - \frac{amoon}{\mathrm{sqrt}(2)}, 0\right]\right], x = 0\,..AU\cdot 149597871000, y = 0\,..AU\right.$$

$$\cdot 149597871000, color = \left[\left[piecewise\left(moontemp(\mathrm{sqrt}(x^2+y^2)) < 273,\right.\right.\right.$$

$$\frac{moontemp(\mathrm{sqrt}(x^2+y^2))}{273}\cdot 0.9, 273 \le moontemp(\mathrm{sqrt}(x^2+y^2)) \le 373,$$

$$\left.\left(\frac{\mathrm{abs}(323 - moontemp(\mathrm{sqrt}(x^2+y^2)))}{50}\cdot 0.9\right), moontemp(\mathrm{sqrt}(x^2+y^2)) > 373, 1\right),$$

$$piecewise\left(moontemp(\mathrm{sqrt}(x^2+y^2)) < 273, \frac{moontemp(\mathrm{sqrt}(x^2+y^2))}{273}\cdot 0.9, 273\right.$$

$$\le moontemp(\mathrm{sqrt}(x^2+y^2)) \le 373, 1, moontemp(\mathrm{sqrt}(x^2+y^2)) > 373,$$

$$\left.\frac{(10^{10} - moontemp(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373}\cdot 0.9\right), piecewise\left(moontemp(\mathrm{sqrt}(x^2+y^2)) < 273, 1, 273\right.$$

$$\le moontemp(\mathrm{sqrt}(x^2+y^2)) \le 373, \left(\frac{\mathrm{abs}(323 - moontemp(\mathrm{sqrt}(x^2+y^2)))}{50}\cdot 0.9\right),$$

$$\left.\left.moontemp(\mathrm{sqrt}(x^2+y^2)) > 373, \frac{(10^{10} - moontemp(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373}\cdot 0.9\right)\right], purple\right],$$

$$numpoints = [1000, 1], lightmodel = none, style = [surface, point], symbolsize = 20, scaling$$

$$= constrained, title = \text{"Circumstellar Moon Habitable Zone"}, labels = [\text{"Distance (m)"}, \text{""}, \text{""}], view$$

$$\left.= 0\,..0.1\right);$$

$$plot3d\left(\left[0, \left[\frac{aplanet}{\mathrm{sqrt}(2)}, \frac{aplanet}{\mathrm{sqrt}(2)}, 0\right], \left[\frac{aplanet}{\mathrm{sqrt}(2)} - \frac{amoon}{\mathrm{sqrt}(2)}, \frac{aplanet}{\mathrm{sqrt}(2)} - \frac{amoon}{\mathrm{sqrt}(2)}, 0\right]\right], x = 0\,..AU\right.$$

$$\cdot 149597871000, y = 0\,..AU\cdot 149597871000, color = \left[\left[piecewise\left(planettemp(\mathrm{sqrt}(x^2+y^2))\right.\right.\right.$$

$$< 273 \textbf{ and } moontemp(\mathrm{sqrt}(x^2+y^2)) < 273, \frac{planettemp(\mathrm{sqrt}(x^2+y^2))}{273}\cdot 0.9,$$

$$planettemp(\mathrm{sqrt}(x^2+y^2)) < 273 \textbf{ and } 273 \le moontemp(\mathrm{sqrt}(x^2+y^2)) \le 373,$$

$$\left(\frac{\mathrm{abs}(323 - moontemp(\mathrm{sqrt}(x^2+y^2)))}{50}\cdot 0.9\right), planettemp(\mathrm{sqrt}(x^2+y^2)) < 273$$

$$\textbf{and } moontemp(\mathrm{sqrt}(x^2+y^2)) > 373, 1, 273 \le planettemp(\mathrm{sqrt}(x^2+y^2)) \le 373,$$

$$\left(\frac{\mathrm{abs}(323 - planettemp(\mathrm{sqrt}(x^2+y^2)))}{50}\cdot 0.9\right), planettemp(\mathrm{sqrt}(x^2+y^2)) > 373$$

$$\textbf{and } moontemp(\mathrm{sqrt}(x^2+y^2)) < 273, 1, planettemp(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } 273$$

$$\leq \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373, \left(\frac{\mathrm{abs}(323-\mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)))}{50} \cdot 0.9 \right),$$

$$\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) > 373, 1 \Big),$$

$$\mathit{piecewise}\Big(\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) < 273 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) < 273,$$

$$\frac{\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2))}{273} \cdot 0.9, \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) < 273 \textbf{ and } 273 \leq \mathit{moontemp}(\mathrm{sqrt}(x^2$$

$$+y^2)) \leq 373, 1, \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) < 273 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) > 373,$$

$$\left(\frac{(10^{10} - \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9 \right), 273 \leq \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373, 1,$$

$$\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) < 273,$$

$$\frac{(10^{10} - \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9, \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } 273$$

$$\leq \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373, 1, \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2$$

$$+y^2)) > 373, \frac{(10^{10} - \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9 \Big), \mathit{piecewise}\Big(\mathit{planettemp}(\mathrm{sqrt}(x^2$$

$$+y^2)) < 273\textbf{and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) < 273, 1, \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) < 273$$

$$\textbf{and } 273 \leq \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373, \left(\frac{\mathrm{abs}(323-\mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)))}{50} \cdot 0.9 \right),$$

$$\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) < 273 \textbf{ and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) > 373,$$

$$\frac{(10^{10} - \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9, 273 \leq \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373,$$

$$\left(\frac{\mathrm{abs}(323-\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)))}{50} \cdot 0.9 \right), \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373$$

$$\textbf{and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) < 273, \frac{(10^{10} - \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9,$$

$$\mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373 \textbf{ and } 273 \leq \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) \leq 373,$$

$$\left(\frac{\mathrm{abs}(323-\mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)))}{50} \cdot 0.9 \right), \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)) > 373$$

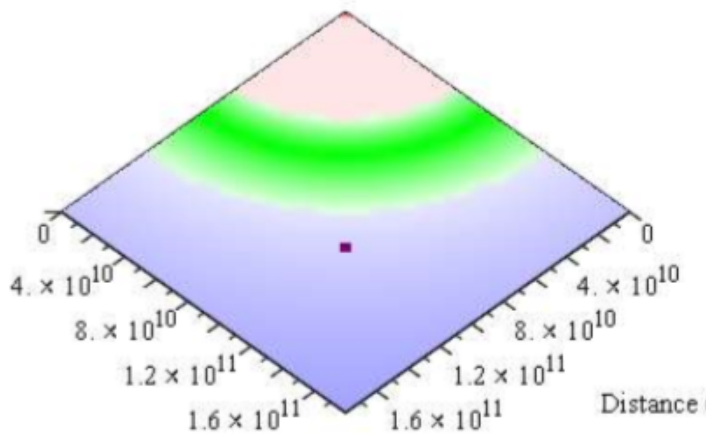
$$\textbf{and } \mathit{moontemp}(\mathrm{sqrt}(x^2+y^2)) > 373, \frac{(10^{10} - \mathit{planettemp}(\mathrm{sqrt}(x^2+y^2)))}{10^{10}-373} \cdot 0.9 \Big) \Big], \mathit{black},$$

$$\mathit{orange} \Big], \mathit{numpoints} = [1000, 1, 1], \mathit{lightmodel} = \mathit{none}, \mathit{style} = [\mathit{surface}, \mathit{point}, \mathit{point}], \mathit{symbolsize}$$

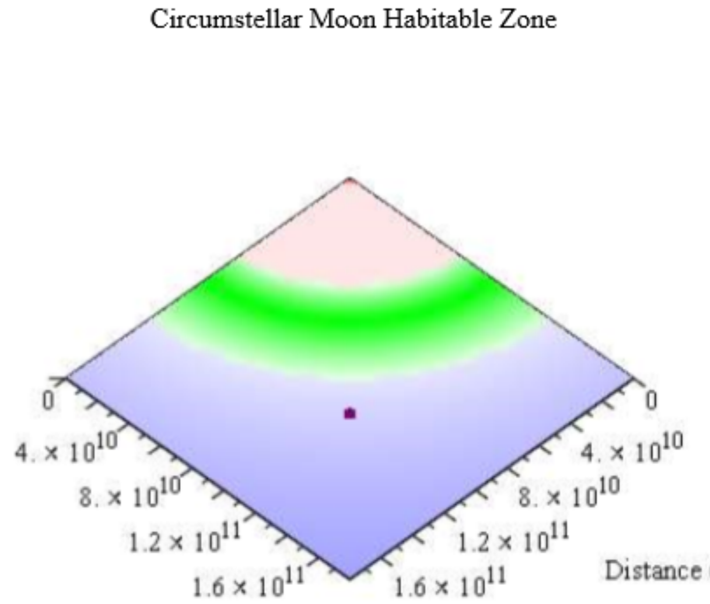
$= 20,\ scaling = constrained,\ title = \text{"Combined Habitable Zone"} \Big);$

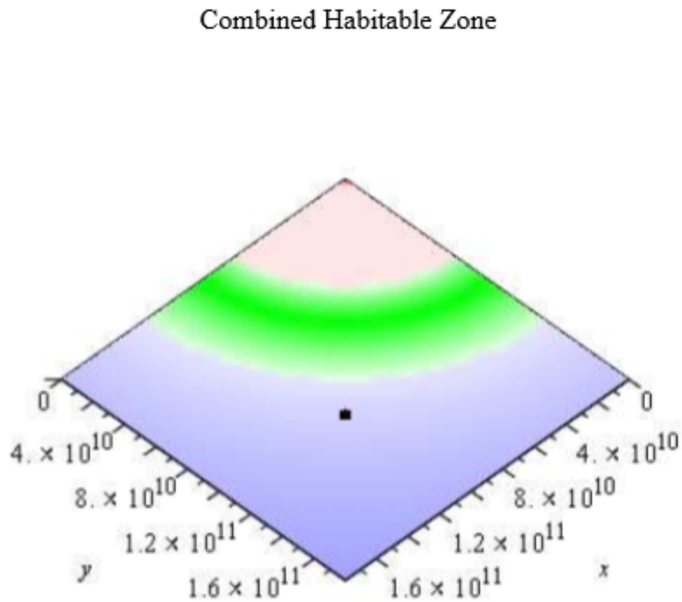
$Planet\ Temperature = 252.10403413834607780\ K$

Circumstellar Planetary Habitable Zone



$Moon\ Temperature = 252.10404621332860862\ K$


Circumstellar Moon Habitable Zone
0
4. × 10^10
8. × 10^10
1.2 × 10^11
1.6 × 10^11
0
4. × 10^10
8. × 10^10
1.2 × 10^11
1.6 × 10^11
Distance (


Combined Habitable Zone
0
4. × 10^10
8. × 10^10
1.2 × 10^11
1.6 × 10^11
y
0
4. × 10^10
8. × 10^10
1.2 × 10^11
1.6 × 10^11
x

Appendix C – Habitable Zone Size with Increasing Tidal Heating

Star Data

Here, you can input or adjust the stellar parameters: these are its radius (in metres), temperature (in Kelvin), and mass (in kilograms)

$Rstar := 695800000$:

$Tstar := 5778$:

$Mstar := 1.9891 \cdot 10^{30}$:

Planet Data

Here, you can input or adjust the planetary parameters: these are its radius (in metres), mass (in kilograms), semimajor axis length of its circumstellar orbit (in metres), and eccentricity of said orbit (unitless):

$Rplanet := 69911000$:

$Mplanet := 1.89813 \cdot 10^{27}$:

$aplanet := 149597871000$:

$eplanet := 0.0$:

Constants

Here are several constants that may be changed. AU is the maximum circumstellar orbital distance along either the x or y axis you wish to evaluate the colour map to (in astronomical units), sigma is the Stefan-Boltzmann constant (in $\frac{W}{m^2K^4}$), G is the gravitational constant (in $\frac{m^3}{kg \cdot s^2}$), k is the second-order Love number of both the planet and moon (unitless), tau is the constant time lag of the tidal bulge (in seconds), A is the albedo of the planet and moon (unitless), and Digits defines how many digits are used in the calculations.

$AU := 1.2$:

$\text{sigma} := 5.670373 \cdot 10^{-8}$:

$G := 6.67384 \cdot 10^{-11}$:

$k := 0.3$:

$\text{tau} := 638$:

$A := 0.33$:

$Digits := 20$:

Calculation

What follows it the code of the calculation. It will return the planet's temperature, the moon's temperature, separate graphs of the circumstellar and circumplanetary habitable zones, and a superimposition of the two graphs.

$$planettemp := (dist) \rightarrow \Bigg(\bigg(Rplanet^2 \cdot 0.52 + \frac{\text{Pi} \cdot Rplanet^2 \cdot (1 - A) Rstar^2 \cdot \text{sigma} \cdot Tstar^4}{dist^2} + \frac{3 \cdot G^2 \cdot k \cdot Mstar^2 \cdot (Mstar + Mplanet) \cdot Rplanet^5 \cdot \text{tau}}{dist^9 \cdot \text{sqrt}(1 - eplanet^2)^{15}} \bigg(\Big(1 + \frac{31}{2} eplanet^2 + \frac{255}{8} eplanet^4 + \frac{185}{16} eplanet^6 + \frac{25}{64} eplanet^8 \Big) - \Big(1 + \frac{15}{2} \cdot eplanet^2 + \frac{45}{8} \cdot eplanet^4 + \frac{5}{16} \cdot eplanet^6 \Big) \cdot \Big(1 + 3\, eplanet^2 + \frac{3}{8} eplanet^4 \Big)^{-1} \bigg) \bigg) \cdot \big(4 \cdot \text{Pi} \cdot Rplanet^2 \cdot \text{sigma} \big)^{-1} \Bigg)^{\frac{1}{4}} :$$

$Planet\ Temperature = evalf(planettemp(aplanet))\ K;$

$$\frac{solve(planettemp(dist) = 273, dist)}{149597871000};$$

$$\frac{solve(planettemp(dist) = 373, dist)}{149597871000};$$

$$Planet\ Temperature = 252.10436353742562190\ K$$

$$-0.85275589227244136957, 0.85275589227244136957$$

$$-0.45678519064999068468, 0.45678519064999068468 \tag{1}$$

$$SizeofRange[0] = 0.85275589227244136957 - 0.45678519064999068468$$

$$SizeofRange_0 = 0.39597070162245068489 \tag{2}$$

$$eplanet := 0.3; \frac{solve(planettemp(dist) = 273, dist)}{149597871000}; \frac{solve(planettemp(dist) = 373, dist)}{149597871000};$$

$$0.3$$

$$-0.85275567203065585967, 0.85275611251337365309$$

$$-0.45677586580430734330, 0.45679451264117701021 \tag{3}$$

$$SizeofRange[0.3] = 0.85275611251337365309 - 0.45679451264117701021$$

$$SizeofRange_{0.3} = 0.39596159987219664288 \tag{4}$$

$$eplanet := 0.7$$

$$0.7 \tag{5}$$

$$\frac{solve(planettemp(dist) = 273, dist)}{149597871000};$$

$$-0.85255815813320833750, 0.85295294104502017184 \tag{6}$$

$$\frac{solve(planettemp(dist) = 373, dist)}{149597871000};$$

$$0.46419146847810086889, -0.44694750104607443875 \tag{7}$$

$$SizeofRange[0.7] = 0.85295294104502017184 - 0.46419146847810086889$$

$$SizeofRange_{0.7} = 0.38876147256691930295 \tag{8}$$