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An Exploration of Black Hole Thermodynamics

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SUMMARY

Black hole thermodynamics concepts, and mathematical models are often utilized to better understand the characteristics of black holes. The fundamentals of black hole thermodynamics involve the four laws of black hole mechanics and the attributes of black holes, including black hole anatomy and surrounding spacetime. One struggle in black hole physics involves quantifying an entropic value to black holes while simultaneously obeying the second and third laws of thermodynamics. To obey these laws, the generalized second law of black hole mechanics can be applied to entropy and Hawking radiation calculations.[1] Stephen Hawking's findings from his revolutionary paper, *Particle Creation by Black Holes*, have changed the field of black hole physics and are worth investigating. Using the aforementioned content, derivations for the pertinent mathematic formulae can be summarized and analyzed. Conducting an in-depth exploration of black hole dynamics, entropy, and Hawking radiation will elucidate the complex nature of black holes.

ABSTRACT

Improvements in technology have enabled researchers to study interstellar phenomena at an exponential rate. Among some of the most enticing of such marvels are black holes. As a result of extensive research and exploration, the progression of black hole thermodynamics has yielded promising interdisciplinary results, particularly in the fields of math and physics. This paper will explore the fundamental concepts pertaining to black holes, and a brief outline of the scientific contributions made by notable theorists. These concepts will be followed by an in-depth discussion concerning the characteristics of black holes and black hole radiation. Derivations of formulae relevant to Hawking radiation, black hole entropy, and classical black hole dynamics will be summarized, as these are imperative to understanding the complex mechanisms that occur within a black hole. Furthermore, these formulae will be applied to pre-existing interstellar phenomena in order to understand the significance of the theoretical concepts related to black holes. Ultimately, the derived formulae will be put to the test using real black holes. A model will also be generated in a Python environment to visually exemplify Hawking radiation for a Schwarzschild black hole. Applications of this research have proven to enhance understanding in the fields of thermodynamics, astrophysics, quantum mechanics, and mathematics. The research and development towards the thermodynamic concepts related to black holes is imperative to elucidate the fundamental laws of physics that govern the forces that occur here on Earth.

Keywords: Black hole, Hawking radiation, entropy, classical dynamics, generalized second law

INTRODUCTION

Studying the thermodynamic properties of black holes has perplexed theoretical physicists for decades. However, Bekenstein has stated that there are many similarities between black hole physics and thermodynamics worth investigating.[2] Most black holes result from a star exhausting its thermonuclear fuel. Ultimately, the unstable core collapses on itself due to intense gravitational forces, resulting in an infinitesimally small volume and infinitely large point of central density called the singularity.[3,4] Alternatively, black holes can have non-stellar origins, perhaps forming from large quantities of interstellar gas that collapse into a supermassive black hole. These black holes are theorized to be at the centre of many observable galaxies, including the Milky Way.[5] An important feature of a black hole is the event horizon, a theoretical boundary that is located outside of the singularity. The event horizon has strong gravitational forces such that it prevents all particles from escaping a black hole. Exploring features of a black hole, such as the singularity and event horizon, will be crucial to developing a holistic understanding of black hole thermodynamics as each of these factors influence particle and spacetime behaviour.

The Four Laws of Black Hole Mechanics

This paper will explore several intriguing phenomena that occur at the event horizon. In order to analyze the thermodynamic properties of black holes, the four laws of classical black hole mechanics must be considered. These laws will be the basis for the complex theories discussed subsequently, such as black hole entropy and Hawking radiation. First, the curvature and coordinates of a black hole and the surrounding spacetime will be formulated based on theories of the event horizon. These mathematical expressions signify relationships between black hole attributes and serve as the foundation for classical black hole dynamics.

In Newtonian physics, the escape velocity v from a body with mass M and radius r, with the standard gravitational constant G, is defined in the equation below.

$$v = \sqrt{\frac{2GM}{r}}$$

(Eq. 1)

For a black hole with a Schwarzschild radius r_s, the escape velocity must be greater than the speed of light c.[6] The Schwarzschild radius is the distance from the singularity of the black hole to the surface. Thus, the previous equation can be modified to:

$$c < v = \sqrt{\frac{2GM}{r_s}}$$

(Eq. 2)

One can define a spherically symmetric vacuum metric in Schwarzschild coordinates as equation 3.[7]

$$ds^2 = \left(1 - \frac{r_s}{r}\right)dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi)$$

(Eq. 3)

Although it is not derived here, this function plots a line element on the surface of a Schwarzschild black hole.[7] A line element is simply a line segment on any given surface. In this equation, r_s is the Schwarzschild radius, r is the distance of the line element from the singularity, ds is the distance of the line segment, and the angles θ and φ are the standard spherical coordinates for three-dimensional space.

Any infinitesimal distance on the surface of a Schwarzschild black hole can be modeled if the angle and radius of the line element is provided. Additionally, the outlined Schwarzschild coordinates require modification into Eddington-Finkelstein coordinates as time reaches infinity at the event horizon of a black hole. This is not accounted for in equation 3. Thus, all spacetime coordinates are singularized at that location because it is the final destination for any particle. Moreover, any particle that reaches the singularity of a black hole cannot escape, and will be trapped there for the duration of its spacetime continuum. Therefore, there are no points in spacetime beyond the singularity. The equation for Eddington-Finkelstein coordinates[8,9] can then be defined as

$$ds^2 = \left(1 - \frac{r_s}{r}\right)dv^2 - 2dvdr - dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi)$$

(Eq. 4)

This entails that the curvature of the black hole diverges significantly towards its center at $r = 0$, the singularity.[10]

Before proceeding with the mathematical derivations, it is pertinent to first understand the different types of theoretical black holes. Stationary black holes are ones that remain at a certain point on a spacetime plane, but exhibit rotational motion and angular momentum, J.[11] Static black holes also remain at a fixed point, but do not rotate.[11] This distinction is important as some attributes, particularly Hawking radiation, are only observed in rotating, stationary black holes.[12]

The singularity theorems proposed by Penrose and Hawking explain the origin of singularities due to intense gravitational effects at the center of black holes.[13] Given the strength of gravity in a black hole, the singularity is a point at $r = 0$ where there is an infinite gravitational well; ultimately, no matter, light, or information can escape.[10] Penrose proposed the idea of the event horizon being analogous to a trapper surface.[14] This is a surface where all vectors are nullified. Essentially, any geometric object with magnitude and direction will cease to exist after entering this trapper surface. Thus, no movement of light can occur. The nullification of vectors is analogous to the entrapment of particles. The force of surface gravity is directed orthogonal to the trapper surface, in the downward direction toward the singularity.[14] This is because any particles or light rays traveling in the direction of a black hole will gradually shift their trajectory toward the singularity.[14] The convergence of exterior light rays in the direction of the singularity is defined as the null geodesic congruence.[15]

Geodesics are curvatures of the shortest path between two points on a surface.[15] Null geodesics are the same curves traced by massless particles, such as photons, where no time is associated with the distance as the particles move at the universal speed limit.[15] They are referred to as null geodesics because a photon has no time or age associated with it as it travels at the speed of light. Therefore, null geodesic congruence is the curvature of a photon converging towards the singularity of a black hole. In order to model the convergence, ρ, of the event horizon, the area has to be defined as a fractional rate of change, δA

$$\rho = \frac{d}{d\lambda} ln\delta A$$

(Eq. 5)

where λ is the affine parameter for the null geodesics.[4,16] An affine parameter is the boundary for an arc length of a null geodesic. The change in the convergence of the congruence can be written as

$$\frac{d}{d\lambda}\rho = \frac{1}{2}\rho^2 + \sigma^2 + R_{ab}k^a k^b$$

(Eq. 6)

where σ is the shear tensor, and $R_{ab}k^a k^b$ is the affine parameter.[17] The shear tensor is defined as a set of multi-linear operations applied to an object falling towards the black hole; where, the surface gravity elicits a force of tension on the material.[17] This effect is not generally observable in everyday objects as the surface gravity on Earth is rather weak, compared to that of a black hole. Given that the surface of a black hole, or any spherical object, cannot fully converge past the singularity, the surface gravity would need to diverge at this point. Here, as $\Delta R_{ab}k^a k^b$ approaches 0 while still being greater than or equal to zero. The affine parameter will still be positive, as a negative will imply a decrease in surface gravity. However, the change in the affine parameter will decrease. Therefore, as the surface shrinks, the stress tensor on any material will approach a magnitude of infinity.[17] Essentially, the surface gravity on an object falling towards a black hole will increase exponentially as distance between the object and singularity decreases. However, once the object reaches the singularity, where surface gravity is theoretically infinite, the particle will start to diverge.[7] This is because the point of singularity is theoretically the end of the spacetime continuum. Ultimately, any massive object that becomes part of the black hole singularity will have its surface gravity elicited outwards.[17] This is because the singularity is the point where all mass is concentrated. Surface gravitational field vectors only act directed towards the outside of the singularity. This explains why surface gravity converges greatly as objects approach the singularity but diverge once they are integrated into the singularity.[7]

Particle Energy in a Black Hole

Now that the gravitational principles of an event horizon have been examined, the exploration of the energy that black holes generate by absorbing particles can be initiated. The first law of thermodynamics states that energy cannot be created or destroyed; hence, the energies of particles that are absorbed by a black hole must be fully utilized.[18] The energies absorbed can be rest energy, kinetic, gravitational, or other forms. Additionally, the energy utilized by a black hole can be expended in the form of Hawking radiation. Before Hawking radiation can be explored, the understanding of the extent of energy absorption by a black hole must be considered. This will involve the conversion of mass to energy, abiding by Einstein's general relativity principles.[19] One must first define the event horizon mathematically, before an interaction between it and a particle with mass m can be explored. The event horizon is often termed the Killing vector field ξ^μ, any vectors entering the field will theoretically become nonexistent on the spacetime continuum.[20] This is due to the impossibility of a particle from exiting the black hole after entry; thus, the Killing field terminates a particle's path entirely, leaving only the historical path. Now the conserved quantity of energy for a particle of mass m can be defined as

$$E = m\dot{x}_\mu \xi^\mu$$

(Eq. 7)

where E is the particle energy at the Killing field and $\dot{x}_\mu = \frac{\xi^\mu}{|\xi|}$, the Killing field vectorized.[21] The energy expression above can then be simplified to

$$E = |\xi|m$$

(Eq. 8)

where the distance between the particle and the singularity tends to the Schwarzschild radius, $r \rightarrow r_s$. The energy equation above indicates that the magnitude of a Killing field is applied to the mass of the particle to determine its energy. As this occurs, the energy expended by the black hole would equate to the mass m. The Killing vector $|\xi|$ is an indication of the particle's path approaching the singularity.[20] This suggests that it has a rest-mass energy consisting of kinetic energy and gravitational potential energy. The particle will possess kinetic energy at any given point while it approaches the Killing field, and the gravitational potential energy will be experienced as it falls closer to the center of a black hole.[20] A black hole's energy extraction capabilities elucidate its potential, given that it can convert all of the energy from any particle that enters the Killing field.

For the remaining sections of this paper, the convention $G = c = k = \hbar = 1$ will be used, as this will simplify the upcoming equations. Considering these variables are constants, and the exact values are not necessary to the further elucidation of formulae and concepts.

Zeroth Law

The zeroth law of black hole mechanics is a rather straightforward one: the surface gravity κ of a stationary black hole is constant over the event horizon.[22] This coincides with the zeroth law of thermodynamics which states that *the temperature of a system in thermal equilibrium is constant*.[18] However, this law only applies to non-rotating, or static, black holes. Black holes that rotate are unable to maintain constant angular momentums, which can skew the surface gravity and make it non-uniform along the event horizon.[23]

First Law

The first law relates the change of mass of a black hole, dM, to the change in the black hole's area dA, angular momentum dJ, and charge dQ.[24] The variable κ represents surface gravity, ω represents angular velocity, and Φ represents electrostatic potential. The relationship is given by

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ$$

(Eq. 9)

This shows the first law of black hole mechanics. It relates the change in a black hole's mass to other features of the black hole.[24] This principle is analogous to the first law of thermodynamics which is a relationship between the energy of a system and the heat and work present in the system.[18] By examining the terms in this equation, it can be theorized that surface gravity plays a role in the temperature of the black hole, given that it impacts the energy of the system. Essentially, the surface gravity κ will impact the energy of the black hole system and must be altered as well.[24] For example, if the black hole were to decrease in angular momentum and virtually stop spinning, the surface gravity would increase to compensate for that energy deficiency.[24] This relationship can be deduced from the equation above as all of these variables are included in the equation, and influential to the change in mass of the black hole. The other terms in the equation signify the changes in energy that can arise due to rotation, J, and electromagnetism, Q.[24]

Second Law

The second law states that *the area of the event horizon of each black hole does not decrease with time*.[22] This also implies that the area of the event horizon of a black hole formed by the merging of two black holes would be greater than the sum of the areas of the event horizons of the two black holes that formed it. The reason for this is that the surface area of a black hole, like the entropy of a system, can only increase. This is analogous to the second law of thermodynamics, that is *the entropy of any isolated system never decreases*.[18] However, the second law of black hole mechanics is more restrictive than the second law of thermodynamics. This is since in the latter, entropy can be transferred between systems by means of heat transfer and mass flow, but since black holes cannot bifurcate, area cannot be transferred between black holes.[25]

Third Law

The third law of black hole mechanics states "it is impossible by any procedure, no matter how idealized, to reduce κ to zero by a finite sequence of operations."[22] If one were to reduce the κ of a black hole by adding particles to it, the angular momentum would increase. κ will decrease as more particles are added. Simultaneously, the mass angular momentum of the black hole will approach the critical ratio $J/M^2 = 1$ for which $\kappa = 0$. This ratio is hypothetically possible; however, it would require an infinite amount of time. This implies that a black hole's surface gravity could never reach zero.[25] This is analogous to the third law of thermodynamics, where *the entropy change for any isothermal process involving perfect crystals approaches zero as temperature approaches absolute zero*.[18] A simplified mathematical proof can be done by re-examining the first law of black hole mechanics. If there is the inclusion of additional energy E, this energy would increase the angular momentum J of the black hole, thus, reducing the surface gravity κ.[26] A black hole that is able to achieve a surface gravity of zero is defined as an extreme black hole, for which the relation between mass, charge, angular momentum is

$$\kappa = \sqrt{M^2 - Q^2 - \frac{J^2}{M^2}}, \kappa \to 0$$

(Eq. 10)

An extreme black hole would also have a temperature of 0 K. However, this is not possible because of the minute amounts of entropy that must be present in a black hole and its surroundings and is purely theoretical.[27] If surface gravity κ does tend towards zero, it is the equivalent of stating that entropy of a system tending towards absolute zero is a well-defined constant, just as the third law of classical thermodynamics states. A well-defined constant means that entropy is not increasing, which is not possible in classical thermodynamics. This showcases the analogous nature of entropy and surface gravity principles in thermodynamics. This can be seen as a comparison between entropy and surface gravity because a black hole's surface gravity is impacted by its energy, not indifferent from entropy in a classical thermodynamic situation.

Hawking Radiation

In Stephen Hawking's paper *Particle Creation by Black Holes,* he demonstrates that at the event horizon of a black hole, particles are emitted while their antiparticles are trapped in the horizon.[28] These antiparticles are ultimately consumed by the black hole, thus decreasing the black hole's mass.[28] This has introduced many interesting notions about the entropy of a black hole. This emission of particles can be perceived as a photon near the horizon that splits into two waves, one with positive frequency, and the other with negative frequency. The wave with negative frequency enters the black hole and decreases the system's mass.[29] Hawking realized that black holes emit particles such as neutrinos and photons. These emitted particles are analogous to radiation emitted from a black body. Black body radiation is the thermal equilibrium radiation emitted from a body dependent on the temperature of that body. However, black hole temperature will be dependent on surface gravity, κ, given by equation below.[25]

$$T = \frac{\kappa}{2\pi} \approx 10^{-6}\left(\frac{M_{\odot}}{M}\right) \qquad \text{(Eq. 11)}$$

where T represents the temperature, κ represents the black hole's surface gravity, M_o is the solar mass, and M is the mass of the black hole. In Hawking's case, T is an analogue to the radiated particles. He also found that temperature is approximately equal to the solar mass over one million times the black hole's mass, in units Kelvin.[3] Hawking made this discovery by first considering the collapse of a Schwarzschild black hole in spacetime.[28] Next, he considered the quantum field in spacetime as time approaches infinity.[25] The calculations showed that as time approached infinity, the particles corresponding to the emissions from a black body at the Hawking temperature are given by:

$$T = \frac{\kappa}{2\pi} \qquad \text{(Eq. 12)}$$

Given that energy is lost due to Hawking radiation, the black hole must progressively lose mass. The time at which a black hole fully evaporates is proportional to M^3 and is approximately equal to:

$$t = 10^{-17} M^3 \qquad \text{(Eq. 13)}$$

This implies that a 10^{15} gram black hole would have a lifetime of about 10^{17} seconds, or approximately 3.17 billion years, while a black hole of solar mass would have a lifetime of 10^{54} times the age of the universe.[18]

The Generalized Second Law

There is great difficulty when applying the second law of thermodynamics to a black hole using classical mechanics. This was explained simply to Jacob Bekenstein during his graduate studies by his advisor John Archibald Wheeler as, "if I drop a teacup into a black hole, I conceal from all the world the increase in entropy".[30] Wheeler had thought that the matter, in this case a teacup, would disappear into a space time singularity and lose all the entropy associated with it.[25] However, Bekenstein worked around this problem and postulated the generalized second law of black hole mechanics in the equations below:

$$S' \equiv S + S_{bh} \qquad \text{(Eq. 14)}$$

$$S' \equiv S + \frac{A}{4} \qquad \text{(Eq. 15)}$$

Bekenstein defined S' as the *generalized* entropy, S as the ordinary entropy outside a black hole, and S_{bh} to be the black hole entropy.[24] When considering the generalized second law, the second law of thermodynamics is not violated as the ordinary entropy is replaced with the general entropy.

However, Bekenstein identified flaws in the generalized second law. If a box with entropy S and energy E is slowly lowered towards a black hole adiabatically, its entropy will be absorbed by the black hole fully, while the energy can be recovered as work.[31] From the second law of thermodynamics, the energy in the box can be expressed as work which means this arbitrary action acts as a Carnot cycle with 100% efficiency.[25] A Carnot cycle is the most efficient engine possible in thermodynamics and is purely theoretical. Quite apparently, this is a violation of the second and third laws of thermodynamics. However, Bekenstein proposed that if the box was lowered in a quasi-static manner, a slow enough process that maintains thermal equilibrium, the box will not get close enough to the horizon to incur the aforementioned phenomenon, meaning the generalized second law is still valid.[25]

A problem arose with the generalized second law after the discovery of Hawking radiation. To compensate for the lost energy from Hawking radiation, according to the conservation of energy and mass-energy equivalence, the black hole must lose mass.[25] However, this violates the ordinary second law of black hole mechanics, since this implies that entropy is also lost, which cannot occur in the closed black hole system.[25] But, according to the black hole entropy equation, the generalized black hole entropy would not decrease from Hawking radiation.[25] This implies that the generalized second law of classical thermodynamics holds for this scenario, but the second law of black hole mechanics does not.[25]

Black Hole Entropy

As described by Bekenstein, entropy is one of the most "abused" terms in physics.[32] By this he refers to the fact that several different measures of entropy have been developed and utilized. However, complications still arise when applying different measures of entropy to a black hole.[32] This is because they are unrefined areas of knowledge. Some researchers even predict that it is favourable to ill-refer the entropic value of a black hole "entropy" as a more accurate description is "entropy-like quantity." This is because the exact location of a black hole's features, such as microstates and charge, are unknown.[33] Nevertheless, in order to define an entropic value of a black hole, one must look at entropy as a measure of disorder or unknown information.[32]

The first approach for a black hole entropy calculation was given by Hawking and Gibbons in 1977.[34] This calculation was analogous to the entropy term in the first law of black hole mechanics. However, this classical approximation of black hole entropy was flawed since the entropy grew too fast for the energy to be a defined value.[25] An alternative approach called entanglement entropy was then developed. This entropic calculation involves computing the trace of the density matrix multiplied by the logarithm of the density matrix as given below

$$S = -Tr\rho \ln\rho \quad \text{(Eq. 16)}$$

where ρ is the density matrix, and S is defined as von Neumann entropy.[35] This computation alone will output a value that diverges. However, if a short distance cut-off—a regularization of the short-distance behaviour of the quantum field—is inserted, the entropy value becomes dependent on the surface area of the event horizon.[35] This is a natural approach to conclude that a black hole's entropy is proportional to its surface area. However, this formula is also dependent on the short distance cut-off which has not been explored thoroughly at this point in time.[25] The most successful calculation of a black hole's entropy is one derived from principles in the field of string theory, which is outside the scope of this exploration.[25]

In summary, assigning an entropic value to a black hole is a rather complex task. Due to varying definitions of entropy and the strange nature of black holes, this problem continues to perplex physicists and does not have a definite solution. Nonetheless, physicists such as Bekenstein have laid a foundation for the entropic value of a black hole and the understanding of this concept will only increase with time.

Black Hole Modeling

Now that the various aspects of black hole mechanics have been explored, they can be model using Python. The specific aspects that will be analyzed and modeled in this section are black hole surface gravity, entropy, temperature, and lifetime. Surface gravity is a geometric concept that has been discussed heavily in this paper.[22] Entropy and temperature closely relate to Hawking radiation and the generalized second law.[36] Lifetime is a good indicator of the evaporation process for a supermassive Schwarzschild black hole, and an interesting attribute worth exploration.[37] Note, each equation is only dependent on the mass of the black hole. All equations used for the following models are provided in the appendix. Although derivations will not be provided in this paper, the equations stem from concepts pertaining to Hawking radiation. Thus, the purpose of this section is to understand and illustrate the relationships between the mass and the various attributes being explored.

Five different black holes were included in this modeling exercise, each was significantly different in mass. These black holes vary in mass but are arbitrary in terms of astronomical location. In this exploration the blackholes at the center of the Milky Way, Andromeda, Sombrero galaxy, the Phoenix Cluster and Messier 87 were compared. Plotting these phenomena on the same graph allows for meaningful relationships with respect to the measured quantities to be derived. The values for each body were found from Georgia State University's online black hole mass database, consisting of approximated interstellar quantities.[38]

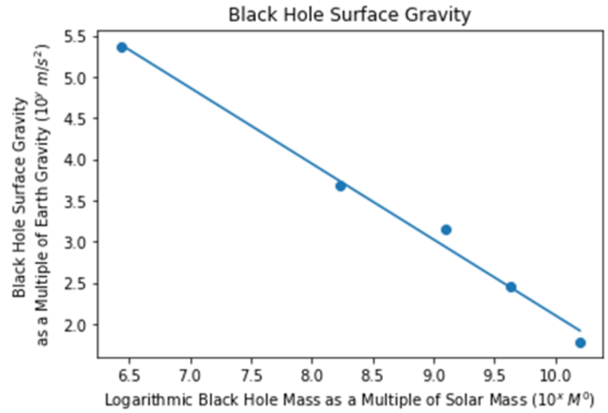

Figure 1. **The surface gravity of five different black holes as a function of solar mass multiples.**

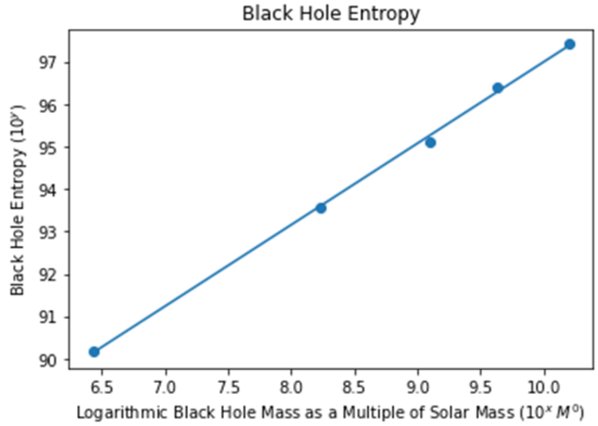

Figure 2. **The entropy of five different black holes as a function of solar mass multiples.**

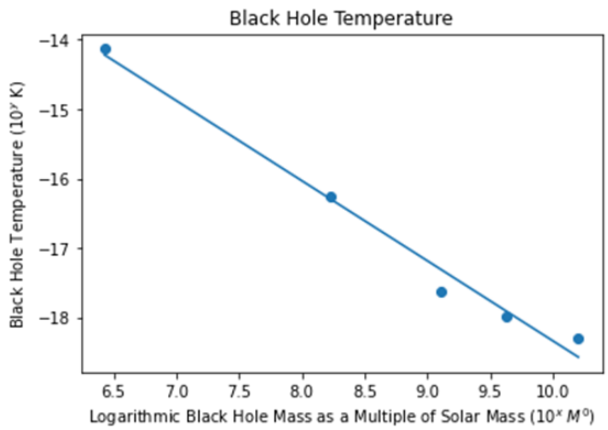

Figure 3. **The temperature of five different black holes as a function of solar mass multiples.**

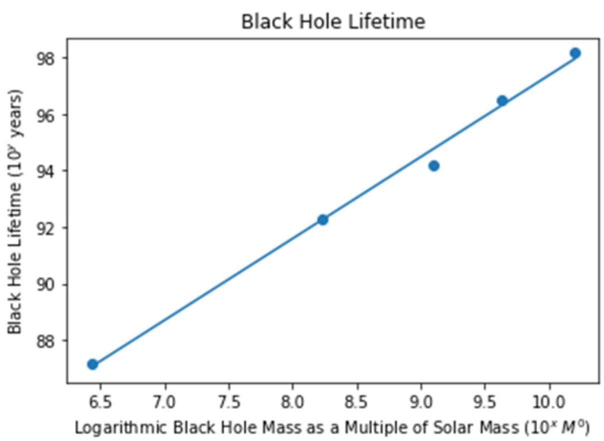

Figure 4. **The lifetime of five different black holes as a function of solar mass multiples.**

Although equations grant the ability to understand relationships, illustrations such as the plots above enable the visualization of the connections between select variables from these functions. The main trends that can be observed from the graphs above are the increase in entropy and lifetime, and the decrease in temperature and surface gravity with increasing solar mass.

Decreases in surface gravity are expected as this relationship was proven earlier in the Third Law section. As the mass and energy of a black hole increases, the surface gravity must decrease to compensate for that increase. The lifetime of a black hole increased as well, which is to be expected as the evaporation of a larger black body will take longer. The most interesting relationship exists between the entropy and the temperature. Conventional thermodynamic theory suggests that increased temperature leads to increases in entropy due to greater sums of kinetic energy distributed in a system, leading to greater degrees of disorder or randomness in the system.[18] However, the opposite appears to be true for a black hole. In order to understand this, the relationship between entropy and temperature as defined by black hole mechanics must be examined. Entropy of a black hole is proportional to the number of Planck-length-sized squares that can be accommodated in the cross-sectional area.[39] Planck length is defined as $l_P \approx 1.62 \times 10^{-35}\ m$

Given that the Planck length is a very small quantity, and how massive black holes generally are, the entropy of a black hole can reach astronomical levels. As black holes get exponentially larger, the entropy will tend towards enormous values.[32] This definition is referred to as the Bekenstein bound and was employed by Stephen Hawking when exploring the temperature for singularities of black holes.[33] Hence, it allows the separation of temperature and energy analysis from conventional thermodynamic theory. The thermal energy inside a black hole is very small as the particles are incredibly dense; hence, they are unable to move and contact one another. Rather, the particles are directed in a very linear path towards the singularity, which prevents them from interacting with one another.[24] Inability to generate movement in a random manner leads to a temperature of nearly absolute zero. As black holes absorb more matter and increase in size, the entropy will increase drastically but the temperature will continue to decrease and approach absolute zero.[25] Essentially, the temperature of a black hole follows the conventional theory of thermodynamics. But, defining the entropy is more difficult as supplemental concepts must be brought in to understand this almost paradoxical relation, and negative correlation between the two variables.

CONCLUSION

This exploration of the literature has elucidated the thermodynamic properties of black holes and provided meaningful insights on the interstellar phenomena. Mathematical equations pertaining to various black hole characteristics and using Python have been summarized and used to model important black hole features. Additionally, the discussion of relevant theories such as classical black hole dynamics, entropy, and Hawking radiation proved fruitful in the exploration of black hole activity. This review summarized the anatomy and geometry of black holes, in addition to the effects they have on the surrounding spacetime. Additionally, the event horizon, singularity, generalized second law and its connection to black hole entropy were important topics thoroughly explored in the literature. Hawking radiation and the non-zero temperature were abundant concepts in the academic literature, thus, explained in this paper. Modeling assisted in contextualizing and illustrating many of these sophisticated concepts; thus, enabling the interpretation of meaningful relationships between black hole characteristics. This was a complex undertaking, but worthwhile given the incredible discoveries made about an obscure cosmological occurrence.

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APPENDIX

Surface Gravity:

$$\kappa = \frac{1}{M}\frac{c^4}{4G^2}$$

Entropy:

$$S = M^2\frac{4\pi G}{\hbar c}$$

Temperature:

$$T = \frac{1}{M}\frac{\hbar c^3}{8\pi k_B G}$$

Lifetime:

$$t = M^3\frac{5120\pi G^2}{\hbar c^4}$$